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Analyses of cracking failure in curved laminated lumber due to transverse stress under a curvature-decreasing moment

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ABSTRACT

Cracking failure of a curved laminated lumber might occur due to transverse stress under a curvature-decreasing bending moment. From both safety and cost perspectives, it is essential to understanding the failure moments and failure modes of curved laminated lumber. While the existing equation to calculate the transverse stress of a curved laminated lumber under a bending moment applied in most literatures is approximate and may cause considerable errors when the initial curvature of beam is small. This is detrimental to the design of curved laminated timber. To solve this problem, we proposed a new equation of bending moment M_c at which the cracking failure is initiated. M_c calculated from this new equation is accurate and larger than $M_{c\text{-}approx}$ calculated from the existing approximate equation. We also derived a novel equation to calculate the minimum ch ($c_{min}h$) below which cracking failure will not occur. Besides, a novel equation to calculate the critical ch ($c_{cri}h$) which represents equal opportunities for cracking and bending failure of the beam to occur was further derived. The model proposed in this article are valuable and practical in the design of curved laminated lumber.

To exert our model to practice, the equations derived in this paper are applied to literature data (*Wood handbook*, 1999) and the results showed that hardwoods have statistically significantly larger average values of three parameters, M_c, $c_{min}h$ and $c_{cri}h$, than softwoods which means hardwoods are more resistant to cracking failure than softwoods. This information is quite useful since lots of laminated lumber for building or furniture are made of hardwoods in Asia.

1. Introduction

Laminated veneer lumber (LVL) is made by multi-layers of veneers in the longitudinal direction and is as an alternative to solid timber or glue-laminated timber. Due to a limitation of large-diameter trees from plantation forests, the availability of large-dimension columns and beams made by sawn timber from logs has decreased. Therefore the topic of R&D and manufacturing of laminated lumber or laminated veneer lumber (LVL) is getting more and more attention. Laminated lumbers are often used as beams, columns, and arches in wood construction and the curved laminated beam is commonly utilized in gymnasiums, churches, museums, bridges and furniture parts. Recent studies about the development of LVL are diverse, ranging from the source of veneer [1,2] to the final application such as seismic design of buildings [3]. Bodig & Jayne [4] offered adequate and detail information for the basic elasticity theory and mechanical properties of laminated lumber composite systems which is invaluable in the design of laminated lumber cost-effectively. Normally, isotropic materials, such as metals and plastics, were fully discussed in materials engineering text [5]. The transverse and longitudinal tensile strengths of isotropic-materials beam are equal. Therefore, the transverse stresses for a beams subjected to bending moment are much smaller than longitudinal bending stresses which means cracking failure will never occur. However, in the case of orthotropic material like wood, the situation is totally different. When a curved beam made of laminated lumber with a rectangular cross-section is subjected to a curvature-decreasing bending moment, transverse tensile stresses will be set up with a maximum at the mid-plane of the beam. Since the transverse tensile strengths perpendicular to the grain are smaller than the longitudinal tensile strengths, the maximum stress of cracking failure is small. When this tensile stress is larger than the tensile strength perpendicular to the grain of wood, cracking of the beam followed by splitting along the grain may occur. Cracking failure is more likely to occur if this coupled with bad glue-line. Since researches related to the failure mode for the curved laminated lumber beam due to transverse stress under a

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curvature-decreasing moment are important, some references related to the curved LVL are illustrated as follows and symbols are listed in Table 1.

An approximate equation applied initial curvature to calculate the transverse tensile stress of a curved laminated lumber under a curvature-decreasing moment was widely cited in textbooks [6,7]. Wu & Furuno [8] also applied the approximate equation to calculate the transverse stresses in curved LVL under a bending moment. Their results showed that the cracking failure occurred at the inner part of laminated lumber made of Masson's pine and fast-growing poplar under a curvature-decreasing moment with initial *ch* of 0.3 where *c* and *h* are, respectively, the curvature and the thickness of beam. The experimental results showed that beams with larger curvature are more prone to cracking and these are consistent with our equation (1.9). Ikuta [9,10] investigated the transverse stress in curved laminated lumber composed of Yezo spruce, Todo fir and oak. Strain gauges were applied to measure the internal strain and stress distributions of the laminated wood, and the results were compared with the stress calculating by the approximate equation and are consistent with our equation (1.6). The results showed that the bending strength of hardwood decreases more slowly than softwood at larger curvatures. Nguyen [11] investigated the effects of curvature on the stresses of a curved laminated beams subjected to bending. The transverse stress was calculated by the approximate equation and was compared with FEM method. Similar results were obtained. In these references, the approximate equation was applied to calculate the transverse stress, however, the error caused by this approximate equation was not mentioned. Actually, this error will be noticeable especially the initial curvature of beam is small.

In this investigation, we derived an accurate cracking failure moment M_c using the tensile strength perpendicular to the grain of wood and final curvature which was expressed as the initial curvature minus the elastic bending curvature change due to the bending moment at cracking. We also derived the equation of the minimum *ch*, below which cracking failure will not occur and the equation of the critical *ch* at which equal opportunities for cracking failure and bending failure to occur. The minimum *ch* and the critical *ch* can constrain the occurrence of cracking failure. Many important data of physical and mechanical properties of commercially important wood species grown in the United States were reported in the Wood handbook [12]. To exert our model to practice, the equations derived in this paper were applied to data from this text, statistical differences in the average minimum *ch* and the critical *ch* of 41 species of softwood and 48 species of hardwood were also compared.

Table 1
Nomenclature.

Notation	
c	Initial curvature of a beam
ch	Dimensionless initial curvature of a beam
$c_{cri}h$	Critical *ch* value at $M_c = M_b$
$c_{min}h$	The minimum *ch* for which cracking can occur. Cracking does not occur for $ch < c_{min}h$
c'	Change of curvature due to bending moment
$c'h$	Dimensionless change of curvature due to bending moment
c'_bh	The $c'h$ at which conventional bending failure occurs
c'_ch	The $c'h$ at which cracking failure occurs
E_L	Young's modulus in the longitudinal direction
I	Moment of inertia of the cross section of a beam
K	$K^2 = (ch - c'_ch)c'_ch$ for $M_c = M$
Q	$Q = (c_{cri}h - c'_ch)$ for $M_c = M_b$
M_b	Bending moment at which the bending failure occurs
M_c	Bending moment at which the tangential cracking failure occurs
$M_{c\text{-}approx}$	Approximate bending moment at which the tangential cracking failure occurs
M	Bending moment that creates a curvature change of c'
h	Thickness of beam
$L_{\text{-}failure}$	Normal strain at which the conventional bending failure occurs. $L_{\text{-}failure} = \sigma_b/E_L$
σ_b	Bending strength in the longitudinal direction
σ_T	Tangential tensile strength perpendicular to grain
ρ	Initial radius of curvature
ρ_f	Final radius of curvature
ρ'	Radius of elastic bending curvature
y	Distance from neutral axis

2. Analyses

2.1. Transverse stress in beams under a pure bending moment

If an initially curved beam is bent to increase its curvature, inward compressive forces are set up in the beam which produces transverse compressive stresses having maximum at the mid-plane. In general, compressive failure will not occur; therefore, this problem is not described in this investigation. On the contrary, if a curved beam with initial curvature c is bent to decrease its curvature (Fig. 1(a)), namely, to straighten the beam, outward tensile forces per unit axial length dF set up by a member of beam with width b of a cross-section and thickness dy set a distance y from the neutral axis (Fig. 1(b)) can be expressed as

$$dF = \left(\frac{1}{\rho + y} - \frac{1}{\rho' + y}\right)\sigma_L b dy \tag{1.1}$$

where σ_L is the bending stress, which is proportional to its distance y from neutral axis, ρ and ρ' are the radii of curvature of the initially curved beam and the elastic bending, respectively, and c' is the elastic bending curvature change of the neutral axis of the beam ($c' = 1/\rho'$ and $c = 1/\rho$), which is proportional to the bending moment M and inversely proportional to the flexural stiffness $E_L I$ and can be expressed as

$$c' = \frac{M}{E_L I} \tag{1.2}$$

where E_L is the longitudinal elasticity and I is the moment of inertia of rectangular cross-section expressed as

$$I = \frac{bh^3}{12} \tag{1.3}$$

After integration, we obtain

$$F = \left(\frac{1}{\rho + y} - \frac{1}{\rho' + y}\right)\frac{M}{I}\int_y^{\frac{h}{2}} by dy = \left(\frac{1}{\rho + y} - \frac{1}{\rho' + y}\right)\frac{M}{2I} b\left(\frac{h^2}{4} - y^2\right) \tag{1.4}$$

$$\sigma_R = \left(\frac{1}{\rho + y} - \frac{1}{\rho' + y}\right)\frac{M}{2I}\left(\frac{h^2}{4} - y^2\right) \tag{1.5}$$

$$\sigma_{R-approx} = \left(\frac{1}{\rho + y}\right)\frac{M}{2I}\left(\frac{h^2}{4} - y^2\right) \tag{1.6}$$

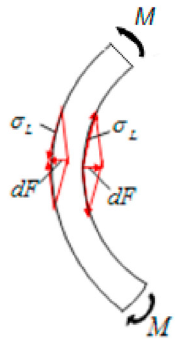
where F is the total outward force per unit axial length, and $\sigma_R = F/b$ is the radial (transverse) stress at distance y from neutral axis.

$$F_{\max} = (c - c')\frac{M}{I}\int_0^{\frac{h}{2}} by dy = (c - c')\frac{Mbh^2}{8I} \tag{1.7}$$

$$\sigma_{R-\max} = (c - c')\frac{Mh^2}{8I} \tag{1.8}$$

where F_{max} is the maximum outward force per unit axial length at the neutral axis. The maximum radial tensile stress ($\sigma_{R\text{-}max}$) can be expressed as F_{max}/b.

If M_c is defined as the cracking failure moment when the transverse tensile stress reaches the tensile strength perpendicular to the grain of air-dried wood, σ_T, and c'_c is the elastic bending curvature change at cracking failure, then M_c can be expressed as



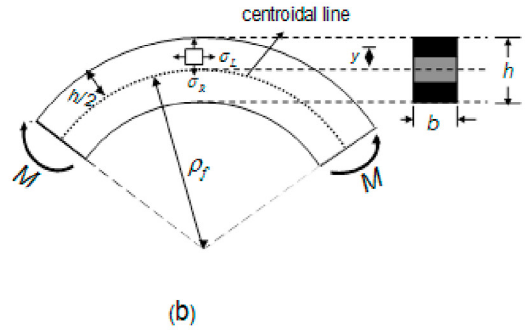


Fig. 1. Originally curved beam under pure bending moment, *M*, to decrease the curvature. (a) Outward tensile forces *dF* of a curved beam (b) Internal stresses in a curved beam.

$$M_c = \frac{8\sigma_T I}{(ch - c_c'h)h} = \frac{2\sigma_T bh^2}{3(ch - c_c'h)} \tag{1.9}$$

$$M_{c-approx} = \frac{2\sigma_T bh^2}{3ch} \tag{1.10}$$

where $M_{c\text{-}approx}$ is the approximate equation for $c \gg c'_c$.

2.2. Determination of critical $c'_c h$ for cracking failure

If we set $M = M_c$, then

$$1 = \frac{M}{M_c} = \frac{(ch - c_c'h)c_c'hE_L}{8\sigma_T} \tag{2.1}$$

if we set

$$(ch - c_c'h)c_c'h = K^2 \tag{2.2}$$

then

$$K = \sqrt{\frac{8\sigma_T}{E_L}} \tag{2.3}$$

where K is a parameter which simplifies the description of elastic bending curvature. From equation (2.2), we obtain the solution of $c'_c h$:

$$c_c'h = \frac{ch - \sqrt{(ch)^2 - 4K^2}}{2} \tag{2.4}$$

The positive sign in front of the square root in equation (2.4) is discarded because a smaller value of the bending moment should be adopted for crack initiation. Furthermore, to obtain the real root of the square root, we must have

$$ch \geq 2K \tag{2.5}$$

$$\because ch - c_c'h = \frac{ch + \sqrt{(ch)^2 - 4K^2}}{2} \geq K \tag{2.6}$$

$$\therefore c_c'h \leq K \tag{2.7}$$

From equations (2.4) and (2).7) we obtain

$$c_{\min}h = 2K \tag{2.8}$$

$$(c_c'h)_{\max} = K \tag{2.9}$$

2.3. Determination of critical $c'_b h$ for bending failure

If a straight beam with a rectangular cross-section is subjected to a pure bending moment *M,* fibers on the convex side are in tension, while those on the concave side are in compression. Stress at the neutral axis is zero, while the maximum compressive stress and maximum tensile stress respectively occur at the outmost concave surface and convex surface. Under a bending failure condition, the breaking moment M_b is expressed as

$$M_b = \frac{2\sigma_b I}{h} \tag{3.1}$$

where σ_b is the bending strength of the wood.

If we set $M = M_b$ then

$$1 = \frac{M}{M_b} = \frac{c_c'hE_L}{2\sigma_b} \tag{3.2}$$

$$c_b'h = \frac{2\sigma_b}{E_L} = 2\varepsilon_{L-failure} \tag{3.3}$$

where $c'_b h$ is the critical $c'h$ when bending failure occurs. $\varepsilon_{L\text{-}failure}$ is the longitudinal strain at the outermost surface of the beam under bending failure. Because the failure mode is determined by the smaller bending curvature, we obtain the following relations:

$c'_b h < c'_c h$ for bending failure and
$c'_b h > c'_c h$ for cracking failure.
If $c'_b h$ is smaller than $c'_c h$, then bending failure occurs, and vice versa.

In the case of $ch < 2K$, if $2\sigma_b/E_L \leq ch$, then bending failure will occur; whereas, if $0 < ch < 2\sigma_b/E_L$, bending failure will not occur in curvature-decreased bending. However, if the bending moment is further increased, bending failure will eventually occur.

2.4. Determination of critical $c_{cri}h$ of a beam with equal opportunities for cracking and bending failure

$$\frac{M_c}{M_b} = \frac{4\sigma_T}{(ch - c_c'h)\sigma_b} \tag{4.1}$$

If we set $M_c = M_b$, then

$$(c_{cri}h - c_c'h) = Q = \frac{4\sigma_T}{\sigma_b} \tag{4.2}$$

where Q is the critical final ch.

$ch > c_{cri}h$ for cracking failure and
$ch < c_{cri}h$ for bending failure.

$$Qc_c^{'}h = K^2 \tag{4.3}$$

$$c_c^{'}h = \frac{K^2}{Q} = \frac{2\sigma_b}{E_L} = c_c^{'}h \tag{4.4}$$

$$c_{cri}h = Q + \frac{K^2}{Q} = \frac{4\sigma_T}{\sigma_b} + \frac{2\sigma_b}{E_L} \tag{4.5}$$

3. Data utilization

In this study, we utilized data related to the mechanical properties of air-dried wood grown in the United States, published in the *Wood handbook*. σ_b, σ_T, E_L, and the air-dried specific gravity were used for the analyses, which included 41 species of softwoods and 48 species of hardwoods. $c_{min}h$ and $c_{cri}h$ in relation to the air-dried specific gravity of wood are discussed.

An analysis of variance (ANOVA) was used in this study. To compare the results of σ_T/σ_b, $c_{min}h$, and $c_{cri}h$ calculated by equations (4.2), (2.8) and (4.5) using data from the *Wood handbook*, the confidence interval (CI) of the difference between the means of hardwoods and softwoods was used. If the p value of the F-test was <0.01, then there was a statistically significant difference between hardwoods and softwoods at the 1% level. Linear regressions of $c_{min}h$ and $c_{cri}h$ related to the air-dried specific gravity were carried out for hardwoods and softwoods. Coefficients of correlation, R^2, were also determined.

4. Results and discussion

From equation (1.10), one can see that the approximate magnitude of M_c is proportional to σ_T, b, and the square of h, and is inversely proportional to ch. But in the accurate equation (1.9), the critical curvature, c'_c, should be introduced. Namely, initial ch is replaced by the final value (ch - c'_ch). At the critical condition of crack initiation $M = M_c$, using equation (2.1) ~ (2.3), c'_ch is calculated with equation (2.4). From this equation, we find that the minimum ch in real cases is $2K$, and the corresponding value of c'_ch is K. We understand that there will be no cracking failure if ch is smaller than $2K$, even when the beam is subjected to a bending curvature c' larger than c. The final curvature (c - c') will become negative, and transverse stress changes from tension to compressive stress. In other words, the smallest values of (ch - c'_ch) in equation (2.6) should be K. This completely differs from the approximate equation (1.10) which has no limitations. Apparently equations (1.9) and (1.10) cannot be used in the case of $ch < 2K$. In cases where bending failure occurs, c'_bh is expressed as $2\sigma_b/E_L$ (equation (3.3)) and $\varepsilon_{L\text{-}failure}$ is the breaking strain under bending failure. If the critical condition of $M_c = M_b$ is considered, we obtain $c_{cri}h$ from equation (4.5) with equal opportunities for cracking and bending failure. In this section, we discuss our case studies of woods grown in the United States using the equations presented in sections §2 and §4. From equation (4.1), we realized that a larger σ_T/σ_b ratio indicates a larger resistance of a beam against cracking failure. Statistically, hardwoods have significantly larger ratios than softwoods at the 1% level.

Taking red oak as an example for hardwood and balsam fir as an example for softwood, the relationship between c'_ch and ch is shown in Fig. 2(a). The maximum $c'_ch = K$ occurred when $ch = 2K$, and it decreased as ch increased. Since M_c is proportional to c'_ch, therefore, M_c decreased as ch increased. The relationship between M_c/M_b and ch is shown in Fig. 2(b). The maximum M_c/M_b occurred when $ch = 2K$, and it decreased as ch increased. M_c/M_b reaches to 1 when ch reaches to 0.201 for red oak and 0.089 for balsam fir, respectively. These special points mean $ch = c_{cri}h$, that is to say, equal opportunities for cracking and

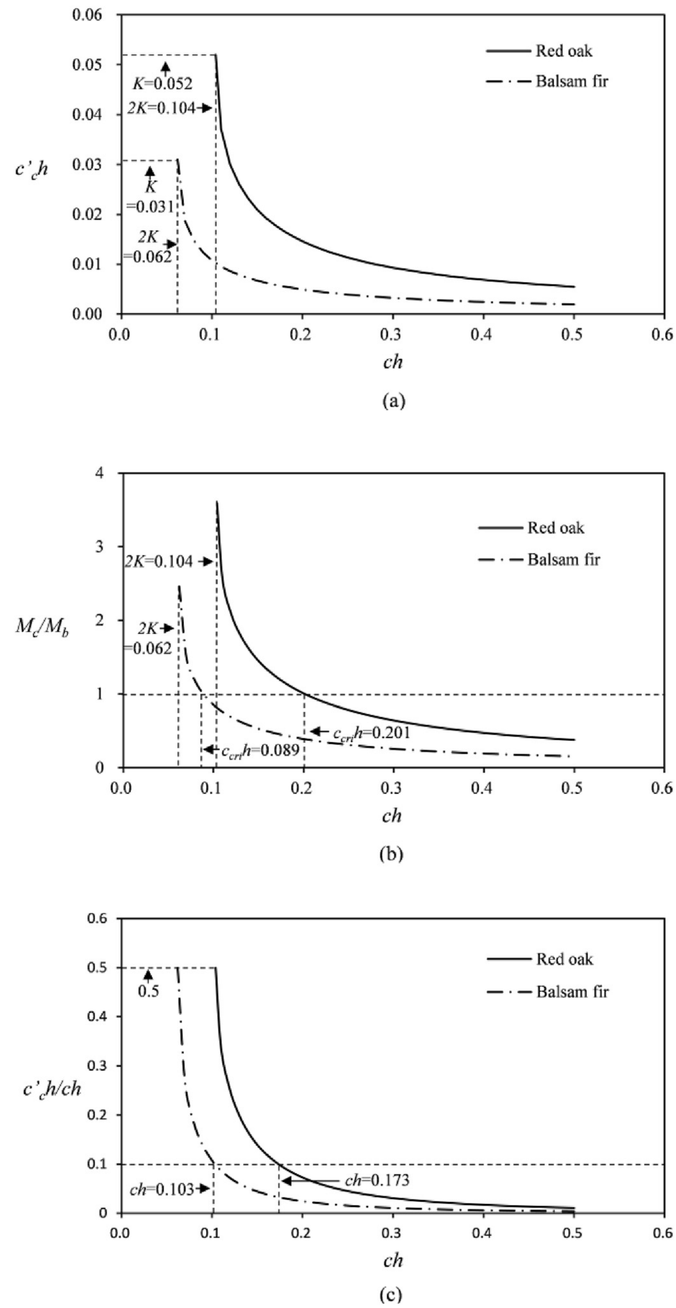


Fig. 2. Relationship between $c'_c\,h$ and ch (a) M_c/M_b and ch (b) and $c'_c\,h/ch$ and ch (c). $c'_c\,h$: calculated by equation (2.4). M_c/M_b: calculated by equation (4.1) Red oak: $E_L = 10300$ MPa $\sigma_T = 3.5$ MPa $\sigma_b = 75$ MPa. Balsam fir: $E_L = 10000$ MPa $\sigma_T = 1.2$ MPa $\sigma_b = 63$ MPa.

bending failure of the beam will occur. Furthermore, cracking failure will happen if its $ch > c_{cri}h$ and bending failure will happen once its $ch < c_{cri}h$. There are differences between $M_{c\text{-}approx}$ calculated according to equation (1.10) and M_c according to equation (1.9) which may cause error. From equations (1.9) and (1.10), $M_{c\text{-}approx}/M_c = 1\text{-}\ c'_ch/ch = 0.5–1$ is obtained. For $ch = 2K$, the estimating error of equation (1.10) will be 50% and becomes negligible with sufficient large $ch >> 2K$. How important would the error be when it comes to reality? What is the threshold of ch below which equation (1.10) should not be used? Suppose an error of 10% was acceptable for the curvature-decreasing case, the error can be expressed as c'_ch/ch. From Fig. 2(c) we find the specific ch is 0.173 and 0.103 for oak and fir, respectively. For oak, the error at $c_{cri}h$ (0.201) is 7.1% < 10%, therefore the approximate equation (1.10) is suitable. On the other hand, for fir, the error at $c_{cri}h$ (0.089) is 14.2% > 10%, the accurate equation (1.9) should be used.

From Fig. 3, one can see that hardwoods have significantly statistically larger $c_{min}h$ values than softwoods at the 1% level. Hardwoods have an average $c_{min}h$ value of 0.117, with a range of 0.084–0.146, while softwoods have an average value of 0.086, with a range of 0.062–0.102. The value of $c_{min}h$ indicates the minimum ch below which cracking failure will not occur.

Similarly, from Fig. 4, we can find that hardwoods have significantly larger values of $c_{cri}h$ than softwoods at the 1% level. Hardwoods have an average $c_{cri}h$ values of 0.235, with a range of 0.139–0.317, while softwoods have an average value of 0.146, with a range of 0.089–0.193. The value of $c_{cri}h$ indicates the critical ch below which M_c will be larger than M_b, and above which M_c will be smaller than M_b. If $M_b > M_c$, then cracking failure occurs; otherwise, bending failure is the dominant mode. The analysis shows that the approximate equation (1.10) is suitable only for $ch > c_{cri}h$.

The combination of glulam can be divided into 2 categories: homogeneous glulam which is made of timber of the same quality and inhomogeneous glulam which is made of timber of different quality. Bending stress at a certain point in a homogenous beam is proportional to the distance between this point and the neutral axis. Bodig & Jayne [4] explained inhomogeneous glulam is a good idea for glulam manufacturing for the costs and effectiveness considerations. It is quite common for a straight glulam that lumber of high elasticity placed at the upper and lower side, and lumber of low elasticity placed in the middle of the beam. High stiffness of inhomogeneous glulam beam can be manufactured assisted by the non-destructive inspection. In Japan, intensive researches and developments of inhomogeneous glulam was composed of two species, such as Japanese cedar with low strength properties placed in the inner layers, and Douglas fir with high strength placed at the outer layers. Other combination such as cedar and oak, i.e., mixing softwood and hardwood laminae in the same gluman is extended [1].

While in the case of inhomogeneous curved laminated lumber, values of M_c, σ_T/σ_b, $c_{min}h$ and $c_{cri}h$ is lower than those of homogeneous, which mean cracking failure is more prone to occur when inhomogeneous curved laminated lumber subjected to a curvature-decreasing bending moment. The lower transverse tensile strength σ_T will cause the decreasing of M_c, $c_{min}h$ and $c_{cri}h$ and cracking failure easier to occur. Hardwoods are more resistant to cracking failure because of a larger σ_T than softwoods. On the other hand, the occurrence of cracking failure will reduce the maximum bending load, and it will reduce the measured bending strength σ_b. Results from Ikuta's bending test [9,10] on curved laminates showed that hardwoods are less prone to cracking failures. Therefore, decrease in bending strength is more moderate than softwoods when the curvature is larger. This issue should be considered in the glulam combination design: the inner part should avoid using low grade lumber to ensure the structural safety for the curved laminated lumber especially when ch is greater than $c_{cri}h$.

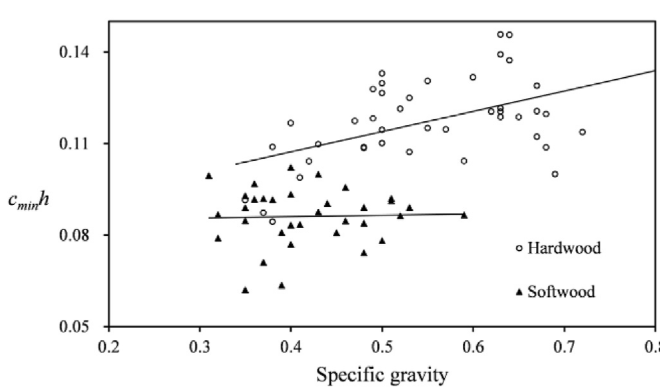


Fig. 3. Relationship between $c_{min}h$ and the specific gravity of air-dried woods. $c_{min}h = 2K$.

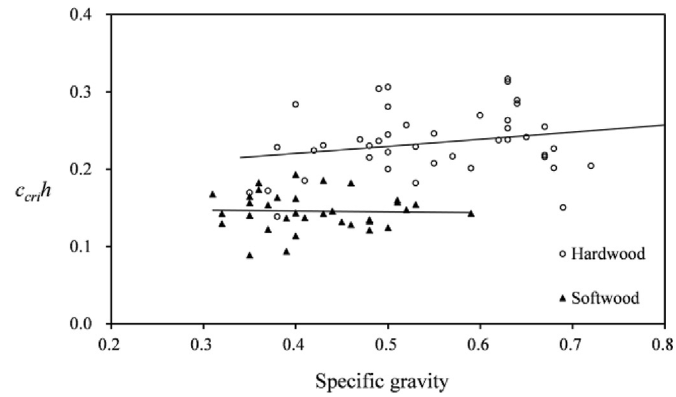


Fig. 4. Relationship between $c_{cri}h$ and the specific gravity of air-dried woods. $c_{cri}h$ is calculated by equation (4.5).

Data from Wood engineering dictionary [13] showed some valuable data in the design of curved laminated lumber such as tensile strength perpendicular to grain is on average one-tenth to one-twentieth of the tensile strength along the grain, and tensile strength in radial direction is about 1.7 times larger than tensile strength in tangential direction due to the existence of wood ray tissue in the radial direction. These data agree with our ideas.

Kollmann & Cote [14] detailed in their text the influence of specific gravity, moisture content, and fiber orientation on the physical properties, elastic modulus, and mechanical properties of wood. Besides, the mechanical properties of wood are also influenced by the presence of knots and the slope of grain produced by sawing method. Abnormal woods like reaction wood and juvenile wood also reduce strength properties of wood. Among these factors, specific gravity has the greatest influence on the properties of wood. The elastic modulus and the strength properties of wood increase with the increase of specific gravity, therefore specific gravity is an important index for evaluating the properties of wood. For the species analyzed here using data from *Wood handbook*, the air-dried specific gravity of hardwoods ranges approximately from 0.34 to 0.72, whereas that of softwoods ranges approximately from 0.31 to 0.59. Hardwoods usually have larger specific gravity than softwoods. However, in the case of relationships between the strength ratio and specific gravity, the tendency becomes unclear for the low R^2 presented. As to the effects of specific gravity on $c_{min}h$ and $c_{cri}h$, we carried out a linear regression analysis. As shown in Figs. 3 and 4, hardwoods present a positive tendency with regression coefficients, R^2, of 0.204 and 0.075, for $c_{min}h$ and $c_{cri}h$, respectively, whereas softwoods present a weak negative tendency with 0.002 and 0.035 for $c_{min}h$ and $c_{cri}h$, respectively. In hardwoods, white oak has the largest values for σ_b/E_L, $c_{min}h$, and $c_{cri}h$, whereas aspen has the smallest values for σ_b/E_L and $c_{cri}h$, and basswood has the smallest value for $c_{min}h$. In softwoods, California red fir has the largest values for σ_b/E_L and $c_{cri}h$, and eastern red cedar has the largest value for $c_{min}h$, whereas black spruce has the smallest values. Based on data from the *Wood handbook*, we found the value of $c_{cri}\ h$ to be about 2-times larger than $c_{min}h$. Nevertheless, gluing hardwood can be more challenging (in particular oak is quite challenging). If wood with high tensile strength perpendicular to grain such as oak is used for a curved laminated-lumber, cracking failure may occur at the bad glue-line. In this condition, glue-ability will become a critical problem.

To visualize the $c_{min}h$ and $c_{cri}h$ of curved beams, the graphic representations for the average $c_{min}h$ and $c_{cri}h$ of hardwoods and softwoods were shown as Fig. 5(a) and (b).

The mechanism of cracking failure of curved laminated lumber under curvature-decreasing moment was clarified in this research. In the design of glulam inhomogeneous curved laminated lumber, to ensure the structural safety, low grade lumber in the inner part should be avoided especially when ch is larger than $c_{cri}h$. Once the grain direction of laminae

Fig. 5. Graphical representations of average values of $c_{min}h$ (■) and $c_{cri}h$ (■). (a) Hardwoods ($c_{min}h$: 0.117, $c_{cri}h$: 0.235) (b) Softwoods ($c_{min}h$: 0.086, $c_{cri}h$: 0.146).

is not parallel to the length of beam, then the bending strength of beam will be largely decreased according to Hankison's formula [4] and the risk of bending failure of the beam will be raised.

5. Conclusions

When a curved laminated wood beam is bent to decrease its curvature, transverse tensile stress will be set up with a maximum value at the mid-plane. If the tensile stress exceeds the tensile strength perpendicular to the grain of wood, cracking failure will occur, followed by splitting along the longitudinal axis. In this analysis, we derived equations to calculate the cracking moment M_c, and $c_{min}h$ below which cracking failure will not happen. More importantly, we introduced an equation for $c_{cri}h$ with equal opportunities for cracking and bending failure. If $ch > c_{cri}h$, then cracking failure is the dominant failure mode. On the other hand if $ch < c_{cri}h$, then bending failure is the dominant mode. The equations of M_c, $c_{min}h$ and $c_{cri}h$ are novel, which are valuable in the design of curved laminated lumber. In real cases, $c_{cri}h$ is about 2-times larger than $c_{min}h$. The approximate equation for calculating the bending moment of cracking failure is suitable only for $ch > c_{cri}h$. Statistically hardwoods have larger mean values of $c_{min}h$ and $c_{cri}h$ than softwoods. This means that hardwoods are more resistant to cracking failure than softwoods.

Authors' contributions

Y.-S.H. conceived the study and developed the theoretical derivations; F.-L.H. prepared the figures; Y.-S.H. and F.-L.H. discussed the results and wrote the manuscript. Both authors gave final approval for publication.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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