Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 56-3 (2023) 367–372

Using information theory to detect model structure with application in vehicular traffic systems

Daniel Lane * Subhradeep Roy *

* Embry-Riddle Aeronautical University, Daytona Beach, FL 32114 USA (e-mail: laned7@my.erau.edu, roys5@erau.edu)

Abstract: As with other complex dynamic systems, modeling traffic systems requires an accurate understanding of how the individual components are coupled, which can be challenging without prior knowledge. Due to a lack of understanding of the nature of interactions between traffic entities, existing models rely on assumptions. The present study evaluates two information-theoretic measures and their ability to quantify and determine the nature of interactions using synthetic data generated from two structurally different traffic simulation models. These measures uncover relationships that describe these interactions without knowing the underlying model, which suggests these measures can be useful in data-driven model discovery.

Keywords: complex systems, information theory, interaction, model identification, traffic modeling

1. INTRODUCTION

The modeling of complex systems, which are composed of many interconnected components, can be challenging, especially when there is little knowledge about the system beforehand.Modeling, simulation, and control design all rely on system identification, which refers to the process of constructing models from observed data. This is a critical step, as inaccurate models may lead to inaccurate predictions and poorly designed controlled devices (Elinger and Rogers, 2019). A variety of approaches are available, ranging from black-box modeling with no prior knowledge of the system structure to estimating parameters under known or assumed model structures (Wang, 2017; Martin et al., 2015; Carnerero et al., 2022; Khosravi and Smith, 2021).

It is particularly relevant to traffic modeling, which is an example of complex systems where group-level features, such as self-organization and phase transition arise from local interactions among vehicles (Tadaki et al., 2013). The structure of a traffic model is not always known, and as a result modelers rely on assumptions. For example, the majority of car-following models assume interactions between vehicles are restricted only to the immediate front vehicle (Treiber et al., 2000), and due to a lack of adequate traffic data, these assumptions are difficult to validate (Aghabayk et al., 2015). If multiple vehicles influence the system dynamics, a traffic model that only includes influences from the front vehicle will be inaccurate. On the other hand, there is an over-fitting problem when the model contains too many terms for vehicles that do not affect the dynamics.

In this regard, system identification techniques can help determine the structure of a model. By collecting data from multiple vehicles traveling in a controlled environment (which eliminates the presence of any confounding factors) and using appropriate data analytic tools it may be possible to detect how many preceding and following vehicles influence a subject vehicle. By focusing on the variables associated with vehicles that directly influence a subject, we can determine the correct model.

Identifying which vehicles influence a subject is even more difficult when nature of interactions are nonlinear. Information-theoretic (IT) metrics may be suitable for this application as they have emerged as a powerful tool to detect relationships among components in complex systems. IT metrics are model-free measures, can capture non-linear relationships and characterize relevant properties of time-series processes and have found diverse applications including human brain activity (Wibral et al., 2014), animal collective behavior (Roy et al., 2019), climate network (Hlinka et al., 2013), and even policy-making (Barak-Ventura et al., 2022).

The application of IT metrics for studying interactions in traffic systems is limited (Roy, 2020; Zhe et al., 2022), despite their popularity in collective behavior modeling. It is critical to validate these metrics in the context of traffic systems since these systems have lane-based maneuvers unlike fish schools or bird flocks. In the present study, we examine the effectiveness of IT tools to characterize interactions among vehicles in the presence of two different traffic conditions - jammed flow (where vehicles experience stop-and-go events) and free flow (where no stop-and-go events occur). Our previous study (Roy, 2020), used one such IT metric with empirical data to identify relationships among vehicles. However, data being collected from human drivers from real experiments, the results show a lot of variability due to drivers' heterogeneity, and the true relationships between vehicles are unknown. In this study, we instead use two different traffic simulation models as toy systems with the known directions of influence in order to test and verify the effectiveness of two IT metrics. Validation of these tools is crucial for their appropriate use in real-world systems and for correctly interpreting the empirical findings. It is expected that the accuracy of these tools in correctly identifying the directionality in toy systems will find confidence in real-world applications to determine what variables need to be included in models, thus eliminating assumptions and enabling data-driven discovery of traffic models. When applied to real-world data, IT analysis may provide a deeper understanding of driver-driver interactions, contributing new insights into human driving be-

10.1016/j.ifacol.2023.12.051

havior that can be applied to autonomous and connected cars to mimic human drivers.

2. METHODS

In this section, we present two well-studied deterministic time-continuous traffic models: the Intelligent Driver Model and the Optimal Velocity Model, which we utilize in the present study, and the information-theoretic tools we use to analyze the resulting data.

2.1 Intelligent Driver Model (IDM)

IDM is a car-following model, where a car is influenced by the distance headway (s) and the relative speed (Δv) with respect to its immediate front car. The car-following behavior is defined in terms of the acceleration function as

$$\frac{dv}{dt} = a\left(1-\left(\frac{v}{v_0}\right)^{\delta}\right) - a\left(\frac{s^{\star}(v,\Delta v)}{s}\right)^2, \tag{1}$$

where v and v_0 are the current velocity and desired velocity, respectively; a is the maximum car acceleration, and δ is acceleration exponent. In the first part of the equation, acceleration decreases from a to zero when approaching the desired velocity, v_0. Interaction with the front car is introduced in the second term through braking phenomenon ($a_{\text{int}} = -a(s^{\star}(v,\Delta v)/s)^2$) where the distance headway s is compared to the desired headway $s^{\star}$. The desired headway is given by

$$s^{\star} = s_0 + \max(0, vT + \frac{v\Delta v}{2\sqrt{ab}}),$$

where s_0 is the minimum allowed gap, T is time headway, and b is a positive quantity denoting comfortable braking deceleration (Treiber and Kanagaraj, 2015). To numerically solve the equations, a fixed update time interval $\Delta t = 0.05$ is considered to integrate. The new speed and new positions are updated as

$$v(t+\Delta t) = v(t) + \frac{dv}{dt}\Delta t \quad \text{and}$$
$$x(t+\Delta t) = x(t) + v(t)\Delta t + \frac{1}{2}\frac{dv}{dt}\Delta t^2.$$

In the case of a stopped vehicle in front, IDM updates may result in a negative acceleration and hence a negative speed in the next time step. This is avoided by imposing the following update rules, i.e., if $v(t) + (dv/dt)\Delta t < 0$, then

$$v(t+\Delta t) = 0 \quad \text{and} \quad x(t+\Delta t) = x(t) - \frac{1}{2}v^2(t)/\frac{dv}{dt}.$$

For simulation, we consider circular roads with a circumference L, and impose a periodic boundary condition that $x \rightarrow x - L$ when $x > L$. In our simulation, we choose typical parameter values (Treiber et al., 2000), as $T = 1.5$, $a = 0.3$, $b = 3$, $\delta = 4$, $s_0 = 2$, and based on the experimental studies (Tadaki et al., 2013), as $L = 314$ meters and $v_0 = 30$ km/hr. We set the initial conditions by placing the cars at equal intervals along the circular track and all the cars are assigned an initial speed of 30 km/hr. In the simulations, vehicle trajectory data are recorded at one second interval for a total of 3000s.

2.2 Optimal Velocity Model (OVM)

OVM is another car-following model, where each vehicle is influenced only by the distance headway s to the vehicle immediately in front, and is formulated in terms of the acceleration function as

$$\frac{dv}{dt} = a_h\left[V(s) - \frac{dx}{dt}\right], \tag{2}$$

where vehicle acceleration is proportional to the difference between optimal vehicle speed $V(s)$ and its own speed. The parameter constant a_h represents drivers' heterogeneity as well as performance profile of individual vehicles (Bando et al., 1995). Here, we assume this parameter is constant for all vehicles. The optimal velocity (OV) function $V(s)$ is a hyperbolic tangent function of headway s given by

$$V(s) = \alpha \tanh[\beta(s - s_o)] + v_o \tag{3}$$

The use of the hyperbolic tangent function allows a given vehicle to accelerate towards a maximum allowed velocity when the headway distance s is sufficiently large. A decrease in headway reduces the optimal velocity to avoid collisions. We use parameter values of $a_h = 1.8$, $\alpha = 5.5$, $\beta = 0.37$, $s_o = 9.1$, and $v_o = 4.9$ based on empirical evidence (Nakayama et al., 2016). To generate the trajectory data, simulation conditions (length of circular track, etc.) are set similarly to those of IDM.

2.3 Information-theoretic (IT) tools:

To measure the directional relationships or coupling between time-series processes, we will consider the notion of transfer entropy (TE) and conditional transfer entropy (CTE), also known as causation entropy (Sun and Bollt, 2014). The idea of pairwise (also called apparent) transfer entropy is to quantify information flow between two time-series processes (Schreiber, 2000; Palus et al., 2001). The idea originates from Shannon entropy, which measures the average amount of uncertainty according to the probability of occurrence of events (Schreiber, 2000). Transfer entropy extends this concept and computes information transfer between two time-series processes X and Y to measure coupling. Specifically, $T_{Y\rightarrow X}$ measures the average amount of uncertainty resolved to predict the future of X from its present, using the additional knowledge of Y at present, and is defined as

$$T_{Y\rightarrow X} = \left\langle \log \frac{p(x_{n+1}|x_n^{(k)}, y_n^{(l)})}{p(x_{n+1}|x_n^{(k)})} \right\rangle, \tag{4}$$

where $\langle . \rangle$ denotes average over all samples, n is the time index, $p(x_{n+1})$ is the probability of x_{n+1}, $p(x_{n+1}|x_n^{(k)})$ is the probability of x_{n+1} conditioned on its past k states $x_n^{(k)}$, and k is the order for the Markov processes. The unit of TE depends on the logarithm base selected, i.e., bits (base 2) or nats (base e). If there is no influence from Y on X, $p(x_{n+1}|x_n^{(k)}, y_n^{(l)}) = p(x_{n+1}|x_n^{(k)})$, and hence $T_{Y\rightarrow X}$ equals zero. Transfer entropy is by construction an asymmetric quantity, so that $T_{X\rightarrow Y}$ is not equal to $T_{Y\rightarrow X}$. This concept identifies the dominant direction of information flow, and thus direction of coupling (Butail and Porfiri, 2019).

When there are more than two time-series processes (X, Y and Z), conditional transfer entropy can be considered. Specifically, conditional transfer entropy $C_{Y\rightarrow X|Z}$ measures the direct influence of Y on X taking Z into account and is defined as

$$C_{Y\rightarrow X|Z} = \left\langle \log \frac{p(x_{n+1}|x_n^{(k)}, z_n^{(m)}, y_n^{(l)})}{p(x_{n+1}|x_n^{(k)}, z_n^{(m)})} \right\rangle, \tag{5}$$

where $p(x_{n+1}|x_n^{(k)}, z_n^{(m)}, y_n^{(l)})$ is the probability of x_{n+1} conditioned on past of x_n, y_n and z_n. When Y does not influence X, the conditional probabilities in the numerator and the denominator are equal, and thus $C_{Y\rightarrow X|Z}$ equals zero. To determine the coupling between time-series variables, this measure can be computed between all pairs, conditioning other variables.

To infer coupling from empirical TE and CTE results measured from a finite number of samples, a statistical significance test using surrogate data can be conducted to determine whether it is statistically different from zero (Roy, 2020). In this paper, we implement the information-theoretic measures and perform a test of significance by using the Java Information Dynamics Toolkit for Matlab (Lizier, 2014). The Kraskov, Stogbauer, and Grassberger method is used to estimate probability distribution functions (PDFs) (Kraskov et al., 2004).

3. RESULTS AND DISCUSSION

Driving in traffic involves continuously gathering and responding to information from the environment and surrounding vehicles, making it an example of a complex system with several coupled components. Discovering traffic models purely from data without prior assumptions and being able to accurately capture driving behavior is a challenging task and a topic of system identification.

To construct a data-driven traffic model for a single-lane scenario, we may start by collecting vehicle-level data (position, speed, etc.) from both the subject vehicle and its neighboring vehicles. In the next step of developing the model of the subject car, the question arises whether to include variables related to all the neighboring cars or just those related to the immediate front car. Generally, traffic models rely on assumptions and are developed by restricting such directional relationships only from the immediate front car to the subject, ignoring any potential influence from the rear car or any other adjacent cars. This study examines whether IT tools can effectively answer this question in the presence of two different traffic conditions - free and jammed flow. If these tools prove effective, we can then determine the minimum number of candidate variables that need to be included in developing a sparse traffic model. Our goal here is to test and validate the effectiveness of the IT tools using two different traffic simulation models as toy systems with known ground truth of directional relationships. Both IDM and OVM are based on the assumption that vehicles respond only to the immediate front car.

We start by simulating vehicles traveling on a single-lane circular track. With this setup, we can record the trajectories for a long time as the vehicles are constrained to remain in the simulation arena and the single-lane traffic eliminates any potential presence of lateral influences from vehicles in the adjacent lanes. The accuracy of PDF estimations requires large samples of data from each car, which is possible within this setup. To simulate jammed traffic with stop-and-go events, we consider the total number of vehicles (N_v) equal to 30, and to simulate free-flow, we consider N_v equal to 15, while keeping the circumference of the track constant between simulations. Car trajectory data are measured along the circumference. Figure 1 shows a visualization of the traffic setup for our simulations. Next, a total of three observables are computed. For i-th car at a given instant, we compute the distance headway between car i and its immediate front car (D^i_F), the distance from its immediate rear car (D^i_R), and the distance the car i travels in the next Δt time interval ($D^i_{\Delta t}$). Consistent with the previous work (Roy, 2020), for our analysis we re-sample the trajectory data with a sampling interval of one second based on the drivers' reaction time.

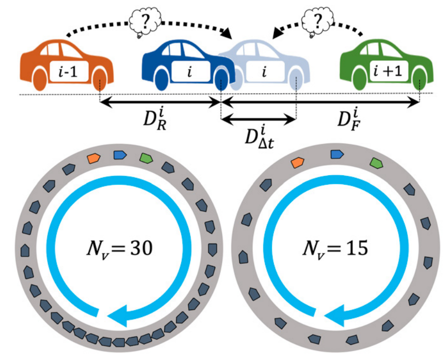

Fig. 1. Visualization of the simulation setup for jammed ($N_v =$ 30) and free ($N_v = 15$) traffic, and the three observables used in the IT analysis to detect the influence of the front and rear cars on a subject car.

As a next step, we employ the IT measures to analyze directional relationships between the time-series variables to detect the influence of the front and rear cars on a subject car. The results of our analysis are presented in Figure 2. Sub-figures in the left column refer to jammed traffic ($N_v = 30$), and those in the right column refer to free traffic ($N_v = 15$). On the plots, the vertical axes represent the TE and CTE values in nats. The cross symbol indicates results that are not statistically significant, meaning that there is no evidence of coupling. Each sub-figure displays the transfer entropy values calculated from the immediate front car and the immediate rear car to a target car, which will be used to infer front-to-target and rear-to-target coupling, respectively.

3.1 Pairwise transfer entropy analyses

We conduct pairwise transfer entropy analyses using data from IDM simulation and the results are shown in the sub-figures (2a and 2b). The dominant direction of coupling between two time-series variables is detected by comparing the level of asymmetry in the TE values (Butail and Porfiri, 2019). For a selected 'target' vehicle, to detect its interaction with the vehicle immediately in front of it and the vehicle immediately behind it, we compute $T_{F\rightarrow T}$ and $T_{R\rightarrow T}$, respectively. For example, for the target vehicle $i = 1$, $T_{F\rightarrow T}$ refers to the pairwise transfer entropy value from its front car ($i = 2$) to itself, which is 0.24 nats (sub-figure 2a) and $T_{R\rightarrow T}$ refers to the pairwise transfer entropy value from its rear car ($i = 30$) to itself, which is 0.18 nats.

For each target vehicle, we observe $T_{F\rightarrow T}$ is greater than $T_{R\rightarrow T}$ (both in free and jammed flow). Thus, TE results correctly infer that the dominant coupling direction is from front to target. However, it fails to capture that there is no influence from the rear car, since $T_{R\rightarrow T} \neq 0$ and the results are identified as statistically significant. This is because as TE fails to differentiate the indirect influences from the direct influences (Sun and Bollt, 2014; James et al., 2016; Bossomaier et al., 2016). Here, indirect influences arise when the distance with the front car changes (as the front car accelerates or decelerates), which influences the target car to update its position accordingly, thus indirectly affecting its distance from the rear car. As the distance from the front car (acts as a 'common driver effect') is

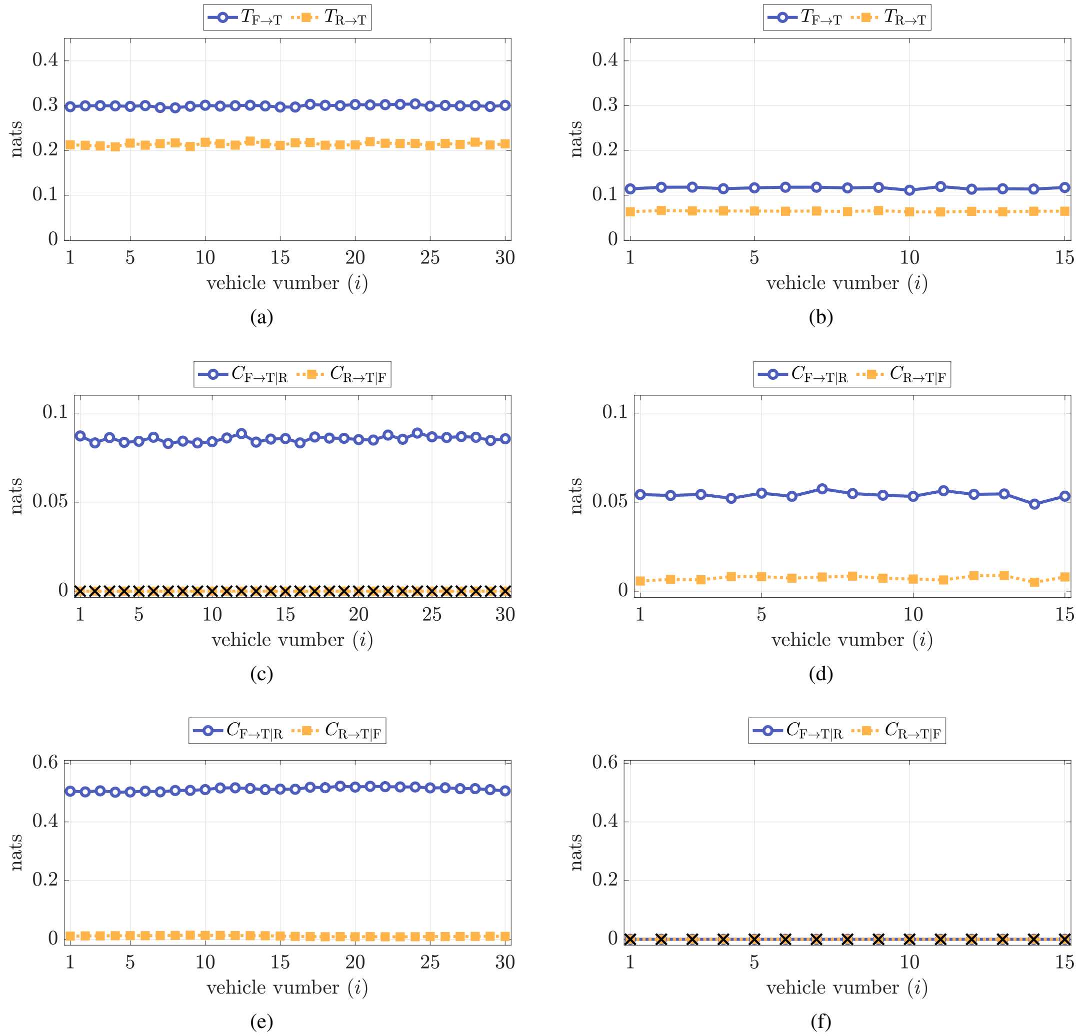

Fig. 2. Pairwise TE (a-b) and CTE (c-d) results computed on simulated IDM data, and CTE results (e-f) computed on simulated OVM data. Sub-figures on the left and the right columns correspond to jammed flow ($N_v = 30$) and free flow ($N_v = 15$), respectively. The symbol cross denotes that the empirical measurements are not statistically different from zero.

not taken into account in $T_{R\to T}$ calculation, pairwise TE detects a false coupling from rear-to-target.

Furthermore, we observe that the transfer entropy values between vehicles in both jammed and free traffic remain nearly constant. This is due to the fact that the simulated behavior of all the vehicles is analogous. Next, we measure the level of asymmetry ($T_{F\to T} - T_{R\to T}$) to examine whether the front car exerts a different level of influence on a target under jammed and free traffic. We calculate this difference and average it across all the vehicles. In the jammed flow, the value is 0.0547 nats, while in the free flow, it is 0.0420 nats. Thus, the asymmetric quantity infers that in jammed traffic, more information is being transmitted from the front car to the target, indicating a stronger front-to-target coupling compared to free-flow.

In summary, the transfer entropy analysis accurately identifies the dominant coupling from the front to the target. It also indicates that the front-to-target influence is stronger in jammed traffic. However, it incorrectly infers a significant coupling from rear to target as it fails to distinguish the indirect dependencies. Next, we examine the CTE measure to see if it can overcome these limitations and also confirm the stronger front-to-target coupling in jammed traffic.

3.2 *Conditional transfer entropy analyses*

Figures 2c and 2d present the results of CTE analysis of IDM data and Figures 2e and 2f present those of OVM data.

CTE analysis of IDM data In both jammed (Figure 2c) and free traffic (Figure 2d), the CTE results correctly identify significant coupling from the front vehicle to every target vehicle. Furthermore, the results corresponding to jammed traffic (Figure 2c) accurately indicate that vehicles are not influenced by the rear cars, as shown by the cross symbols since $C_{R\to T|F}$ values are not statistically different from zero. The results are consistent with how interactions are modeled in IDM, which only includes influence from the immediate front car.

However, in the free traffic (Figure 2d), rear-to-target results appear to be significant, although the values are almost close to zero, with an average of 0.0071 nats computed over all 15 vehicles. Based on this finding, we comment that inferring coupling requires combined knowledge of both the significance test along with the actual values of the CTE results. In this case, despite rear-to-target coupling being identified as statistically significant, its value of almost near zero indicates that such couplings are not truly present, and thus can be ignored.

Consistent with the TE results, we observe that there is not much variation in the CTE values between vehicles in either jammed or free traffic because of the analogous simulated behavior. We further observe an increasing trend in the $C_{F \to T|R}$ values in the jammed flow when compared with the free flow. These findings are in agreement with our pairwise TE results, which also indicate greater front-to-target influence in jammed traffic. This may be a consequence of an increased number of ‘stop-and-go’ events, where a vehicle is forced to stop during a jam. In information theory, there is a high degree of information associated with less likely events. During the entire span of the simulation, these ‘stop-and-go’ events occur less frequently, and during the occurrence of such events, a higher amount of information is transmitted from the front car to the target, which is reflected in the CTE values. Since ‘stop-and-go’ events are not observed in the free flow, as all the vehicles move smoothly, $C_{F \to T|R}$ values decrease. We further investigate the basis of these results in the below subsection 3.2.3.

CTE analysis of OVM data According to the results of the CTE analysis of the OVM data (Figure 2e), it correctly identifies that the significant coupling is from front-to-target and no significant coupling from rear-to-target (with near zero values) in the presence of jammed traffic. Interestingly, in the free flow (Figure 2f), the CTE results indicate that neither the front nor the rear is influencing the target as a lack of statistical significance is observed across all the vehicles in either direction. To determine whether front-to-target coupling is truly absent, we next examine the models.

Examining interaction levels in the simulation models OVM incorporates interactions through the optimal velocity function as given in equation 2, which is determined by the vehicle’s distance headway, s. The OV function (equation 3) with parameters from our simulation as a function of s is shown in Figure 3a (black solid line), which demonstrates that a vehicle will maintain its allowed maximum speed if its distance headway exceeds a threshold. When headway decreases below the threshold, interactions occur as the following vehicle responds by reducing its speed, which is determined by the OV function. To determine the interaction regimes associated with free and jammed traffic corresponding to our simulations, we compute the headway of all the vehicles as a function of time. The vertical lines represent the mean headway (μ) averaged over all vehicles and over the entire simulation time (dashed line for jammed flow, dotted line for free flow), and shaded regions indicate one standard deviation (σ). For jammed flow, $\mu = 10.47$ and $\sigma = 4.67$; and for free flow $\mu = 20.93$ and $\sigma = 0.046$. We notice that the free flow vehicles follow maximum desired speed and are thus not influenced by their immediate front cars, hence there is no interaction. This illustrates the observed CTE results, which accurately revealed the lack of coupling from the front car (Figure 2f). We further observe that the jammed regime is well within the interaction region of the OV function, and thus CTE correctly identified influence from the front car (Figure 2e).

Similarly, we examine the IDM in order to verify the CTE results. Different from the OVM, IDM incorporates interaction through braking term (a_{int} in equation 1), and is shown in Figure 3b. As can be seen from the mean and standard deviation ($\mu = 10.47$ and $\sigma = 8.35$ for jammed flow; and $\mu = 20.93$ and $\sigma = 0.0008$ for free flow), interactions among vehicles are always present in our IDM simulations, but are weaker for

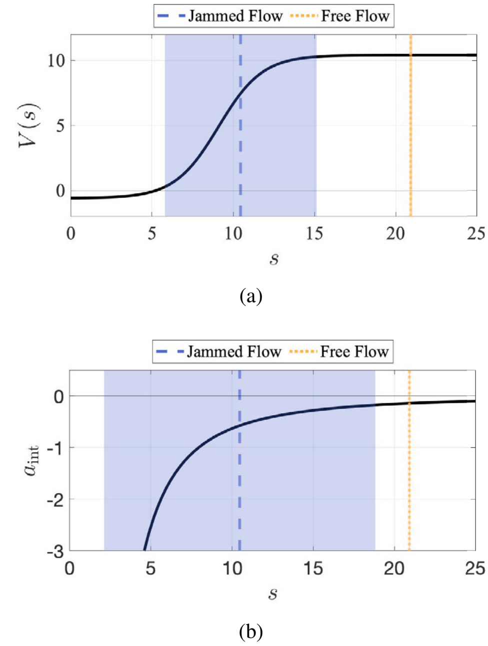

Fig. 3. The interaction regimes of free and jammed traffic in (a) OVM and (b) IDM

the free flow. Thus, the results of the CTE are consistent with these observations. When the models are known beforehand, observing the interaction regimes is easier. With IT tools, however, even without knowledge of the underlying model, we can gain an understanding of the nature of interactions and isolate the interaction regimes.

4. CONCLUSIONS AND FUTURE WORK

Because the models used in our study are structurally different, they incorporate interactions differently. Nonetheless, CTE analysis detects and interprets interactions correctly and thus provides useful insights into model structure. Using the present results from IT measures, it can be inferred that only the variables associated with the immediate front car should be used to identify models, whereas variables from the remaining cars may be ignored. It is evident from our analysis that the drivers’ simulated behaviors are analogous, and as such, less variation is observed in IT measures. When compared to our previous study (Roy, 2020), which uses empirical data, it demonstrates that real-world human driving behaviors are heterogeneous. In some instances, there is directional influence from both front and rear cars on a target, and the values vary significantly between drivers (Roy, 2020).

Furthermore, the present study compares both TE and CTE and concludes that CTE can distinguish indirect influences better than TE. By comparing and validating the IT measures, the present work opens up the possibility of applying them in future empirical studies. The CTE analysis requires conditional independence test on all the potential variables which is difficult to conduct with small sample sizes (which is mostly true for real-world traffic data). In such scenarios, the pairwise TE measures can still be used to determine the dominant direction of coupling, and thus inferring the interaction network. By identifying the correct interaction network, we can identify the correct model structure, which may further help to identify the library of candidate variables for model discovery (Brunton et al., 2016; Champion et al., 2019; Kaheman et al., 2020). A number of questions still to be addressed, including whether IT measures are effective when there is insufficient data or significant noise in the system, when both front and rear coupling are present (Hossain et al., 2021), and whether IT measures can be combined with model discovery algorithms. In our future work, we will expand the framework to incorporate empirical data obtained from our laboratory setup. This will allow us to gather a large sample of data, as we have recognized that the IT tools are data-hungry methods.

ACKNOWLEDGEMENTS

This work was partially supported by National Science Foundation (CMMI- 2238359) and by Embry-Riddle Aeronautical University's Faculty Innovative Research in Science and Technology (FIRST) Program.

REFERENCES

Aghabayk, K., Sarvi, M., and Young, W. (2015). A state-of-the-art review of car-following models with particular considerations of heavy vehicles. *Transport Reviews*, 35(1), 82–105.

Bando, M., Hasebe, K., Nakayama, A., Shibata, A., and Sugiyama, Y. (1995). Dynamical model of traffic congestion and numerical simulation. *Physical Review E*, 51(2), 1035.

Barak-Ventura, R., Marin, M.R., and Porfiri, M. (2022). A spatiotemporal model of firearm ownership in the United States. *Patterns*, 100546.

Bossomaier, T., Barnett, L., Harre, M., and Lizier, J.T. (2016). An introduction to transfer entropy. *Cham: Springer International Publishing*, 65–95.

Brunton, S.L., Proctor, J.L., and Kutz, J.N. (2016). Discovering governing equations from data by sparse identification of nonlinear dynamical systems. *Proceedings of the National Academy of Sciences*, 113(15), 3932–3937.

Butail, S. and Porfiri, M. (2019). Detecting switching leadership in collective motion. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 29(1), 011102.

Carnerero, A.D., Ramirez, D.R., and Alamo, T. (2022). State-Space Kriging: A Data-Driven Method to Forecast Nonlinear Dynamical Systems. *IEEE Control Systems Letters*, 6, 2258–2263.

Champion, K., Lusch, B., Kutz, J.N., and Brunton, S.L. (2019). Data-driven discovery of coordinates and governing equations. *Proceedings of the National Academy of Sciences*, 116(45), 22445–22451.

Elinger, J. and Rogers, J. (2019). Influence of noise on information theoretic causality measures for system identification. In *American Control Conference*, 5648–5654.

Hlinka, J., Hartman, D., Vejmelka, M., Runge, J., Marwan, N., Kurths, J., and Palus, M. (2013). Reliability of inference of directed climate networks using conditional mutual information. *Entropy*, 15(6), 2023–2045.

Hossain, M.A., Ariful Kabir, K.M., and Tanimoto, J. (2021). Improved car-following model considering modified backward optimal velocity and velocity difference with backward-looking effect. *Journal of Applied Mathematics and Physics*, 09, 242–256.

James, R.G., Barnett, N., and Crutchfield, J.P. (2016). Information flows? A critique of transfer entropies. *Physical Review Letters*, 116(23), 238701.

Kaheman, K., Kutz, J.N., and Brunton, S.L. (2020). SINDy-PI: a robust algorithm for parallel implicit sparse identification of nonlinear dynamics. *Proceedings of the Royal Society A*, 476(2242), 20200279.

Khosravi, M. and Smith, R.S. (2021). Nonlinear System Identification With Prior Knowledge on the Region of Attraction. *IEEE Control Systems Letters*, 5(3), 1091–1096. doi: 10.1109/LCSYS.2020.3005163.

Kraskov, A., Stogbauer, H., and Grassberger, P. (2004). Estimating mutual information. *Physical Review E*, 69(6), 066138.

Lizier, J.T. (2014). JIDT: An information-theoretic toolkit for studying the dynamics of complex systems. *Frontiers in Robotics and AI*, 1, 11.

Martin, C.A., Deshpande, S., Hekler, E.B., and Rivera, D.E. (2015). A system identification approach for improving behavioral interventions based on social cognitive theory. In *2015 American Control Conference*, 5878–5883.

Nakayama, A., Kikuchi, M., Shibata, A., Sugiyama, Y., Tadaki, S., and Yukawa, S. (2016). Quantitative explanation of circuit experiments and real traffic using the optimal velocity model. *New Journal of Physics*, 18(4), 043040.

Palus, M., Komarek, V., Hrncir, Z., and Sterbova, K. (2001). Synchronization as adjustment of information rates: Detection from bivariate time series. *Physical Review E*, 63(4), 046211.

Roy, S. (2020). Quantifying interactions among car drivers using information theory. *Chaos: An Interdisciplinary Journal of Nonlinear Science*, 30(11), 113125.

Roy, S., Howes, K., Muler, R., Butail, S., and Abaid, N. (2019). Extracting interactions between flying bat pairs using model-free methods. *Entropy*, 21(1), 42.

Schreiber, T. (2000). Measuring information transfer. *Physical Review Letters*, 85(2), 461.

Sun, J. and Bollt, E.M. (2014). Causation entropy identifies indirect influences, dominance of neighbors and anticipatory couplings. *Physica D: Nonlinear Phenomena*, 267, 49–57.

Tadaki, S.I., Kikuchi, M., Fukui, M., Nakayama, A., Nishinari, K., Shibata, A., Sugiyama, Y., Yosida, T., and Yukawa, S. (2013). Phase transition in traffic jam experiment on a circuit. *New Journal of Physics*, 15(10), 103034.

Treiber, M., Hennecke, A., and Helbing, D. (2000). Congested traffic states in empirical observations and microscopic simulations. *Physical Review E*, 62(2), 1805.

Treiber, M. and Kanagaraj, V. (2015). Comparing numerical integration schemes for time-continuous car-following models. *Physica A: Statistical Mechanics and its Applications*, 419, 183–195.

Wang, Y. (2017). A new concept using LSTM Neural Networks for dynamic system identification. In *2017 American Control Conference*, 5324–5329.

Wibral, M., Lizier, J., Voler, S., Priesemann, V., and Galuske, R. (2014). Local active information storage as a tool to understand distributed neural information processing. *Frontiers in Neuroinformatics*, 8, 1.

Zhe, P., Xu, Q., Runlin, Z., Parkinson, S., and Schoeffmann, K. (2022). Eye movement based information system indicates human behavior in virtual driving. *bioRxiv*.