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# A Capture Strategy for Multi-Pursuer Coordination Against a Fast Evader in 3D Reach-Avoid Games

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**Abstract:** This paper proposes a capture strategy for a team of pursuers to capture a fast evader with uncertain dynamics in a 3D reach avoid game. The strategy involves coordinating the agents motion to spread out and cut off all possible routes to the evader's target. These routes are obtained by estimating the evader's reachable set using mixed monotone reachable set theory. The reachable set is used to determine a capture surface, over which embedded guidance reference points are provided for the pursuers through 2D coverage. The capture strategy is demonstrated via simulation. Results suggest that capture performance improves with an increase in pursuer team size. Further, the strategy is able to outperform a pure-pursuit strategy when there is a sufficient number of pursuers to fully cover the obtained capture surface.



*Keywords:* Networked Systems, Robotics, Pursuit-Evasion Games

## 1. INTRODUCTION

Reach-Avoid (RA) games are a form of differential game in which an evader attempts to reach a target set while avoiding other players. RA games have applications in robotics, defense, and surveillance. Optimal strategies in RA games are known to be found through the Hamilton-Jacobi-Isaacs equation. However, these solutions suffer from the curse of dimensionality and are intractable to compute for games with increasing number of players. We are interested in problems where multiple pursuers coordinate their motion to engage a faster evader. Much of the work related to RA games deals with pursuers that are equal to or faster than the evader, however, there is some work examining faster evaders. In Rivera-Ortiz and Diaz-Mercado (2018) and Davydov et al. (2021), 2D RA games are formulated as control problems and laws found that are superior to pure pursuit. In Garcia and Bopardikar (2021) a control strategy is found for a team of pursuers encircling a fast evader. Similarly, RA game research considers pursuers and evaders in $\mathbb{R}^2$, but recently there have been some results involving agents in $\mathbb{R}^3$. Notably, Garcia et al. (2020) finds solutions to the HJI equations for a small number of players. There is limited work extending results from 2D to 3D because of the added computational complexity of the optimal solutions. Additionally, suboptimal solutions in 3D suffer from a lack of unique solutions stemming from non-convexity which results in an inability to reason about controller performance.

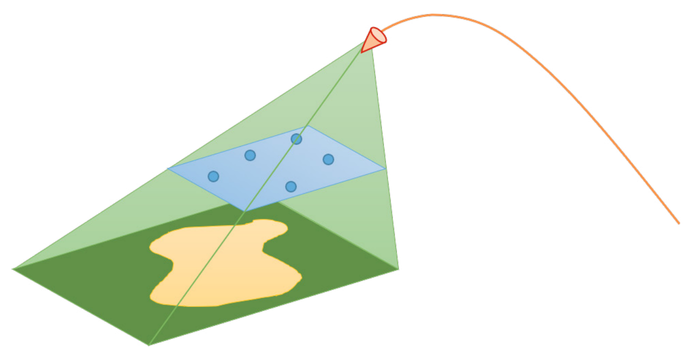

Fig. 1. Capture strategy: An evader (red cone) attempts to reach an unknown goal set (yellow region) within the allotted time. A pursuer team (blue spheres) must coordinate their motion over a capture surface (blue plane) embedded in the evader's reachable set (green volume) which is estimated based on an uncertain evader model and past trajectory information (orange curve).

In this work, we propose a capture strategy deployable on an arbitrarily sized team of slow pursuers to attempt capture a faster evader. By coordinating a team of slower pursuers, a faster evader can be cut off from reaching its target set (see Fig. 1). This work leverages principles from coverage control and mixed monotone reachable set theory to extend approaches presented in Khrenov et al. (2021) from 2D engagements to 3D engagements. We propose a

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controller for a team of 2D embedded pursuers to defend a surface as in Rivera-Ortiz et al. (2020) and capture a 3D ballistic evader with additive uncertainty. The contributions of this work are threefold. We first extend prior scalable strategies for capture in 2D to 3D engagements and consider a ballistic evader model. We then provide a mixed monotone decomposition function for a ballistic trajectory with uncertainty arising from initial conditions and bounds on expected aerodynamic forces. To account for discontinuities that arise from uncertainty, we extend coverage control approaches by using a virtual domain that allows for coverage distributions to be achieved even when real domain boundaries are discontinuous. The ability of the strategy for a team of slow pursuers to achieve capture against a fast evader is explored through Monte Carlo simulation.

The structure of this paper is as follows. We present the reach avoid game problem addressed in this work in Section 2. In Section 3, we discuss the characterization of the evader's dynamics, and estimate its reachable set using mixed monotone theory. We adapt Lloyd's algorithm to provide a capture strategy for the pursuers in Section 4. In Section 5, we describe the validation of results through the simulation framework. Concluding remarks are provided in Section 6.

## 2. PROBLEM DESCRIPTION

We consider the reach-avoid (RA) variant of a pursuit-evasion game, wherein a single evader attempts to travel to (reach) a goal set while avoiding a team of $N > 1$ coordinated pursuers. Due to finite energy budget considerations, we focus on finite-time games, where there exists an upper bound for game conclusion $T < \infty$. If the evader reaches its goal set before this time while avoiding capture, then it wins. Otherwise, the pursuers win.

This work considers the specific case of embedded two-dimensional pursuers and a three-dimensional aerodynamic evader. The pursuer agent dynamics are assumed to be of the single integrator form:

$$\dot{x}_i = u_i \tag{1}$$

where $x_i \in \mathcal{M} \subseteq \mathbb{R}^2$ is the $i^{\text{th}}$ agent's planar position embedded in $\mathbb{R}^3$, $i = 1, \ldots, N$. The velocity of the agents are bounded, such that $\|u_i\| \in [0, v_p]$. The evader's dynamics are unknown, but characterized by a series of typical trajectories, and its maximum velocity is known to be significantly greater than that of the pursuers, i.e., $v_e \gg v_p$. The evader's position is denoted $x_e \in \mathbb{R}^3$. Capture is defined as any member of the pursuer team coming within $\epsilon$ (the capture radius) of the evader, such that

$$d(x_i, x_e) \leq \epsilon$$

where $d(p, q)$ is the Euclidean distance between the evader position in $\mathbb{R}^3$ and the planar position of the pursuer embedded in $\mathbb{R}^3$.

In two-dimensional RA games, one strategy is to form a defense manifold that partitions the evader's reachable set into a set containing the evader and a set containing the target. The pursuers coordinate defense along this partition until the end of the game as in Khrenov et al. (2021). In 3D, the equivalent defense manifold is a 2D surface, which the pursuers defend. To study this situation we formulate this problem as an unpredictable aerodynamic evader with an unknown trajectory, opposing a team of embedded 2D pursers attempting to capture the evader.

## 3. EVADER MODELING AND REACHABLE SET ESTIMATION

In this section we provide a reachable set estimation based on a mixed monotone theory given an uncertain evader model. We begin discussion with the model description of the evader.

### 3.1 Ballistic Evader Model

In this differential game, the evader's trajectory and dynamics are not explicitly known. However, the evader can be characterized by an estimate of its dynamics and bounds on its performance capabilities. Given a set of trajectories that characterize the evader's movement, bounds on its reachable set can be estimated. We focus on a ballistic evader with aerodynamic disturbances, for which the following dynamics are appropriate

$$\begin{bmatrix} \dot{x}_e \\ \dot{v}_e \end{bmatrix} = \begin{bmatrix} v_e \\ 1/m(-T||v_e||v_e + N(v_e \times w) - mg \end{bmatrix} \tag{2}$$

where $x_e \in \mathbb{R}^3$ denotes the coordinates of the evader, $v_e \in \mathbb{R}^3$ denotes the velocity of the of the evader, and $w \in \mathbb{R}^3$ encodes the bounded uncertainty, modeled as spin, with $\langle v, w \rangle = 0$. The constants $T$ and $N$ are associated with the drag and lift, respectively, and $g = [0, 0, g]^\top$.

The spin uncertainty can have drastic effects on the trajectory of the evader. In the sequel, we will use mixed monotonicity to reason about all the points that the evader could reach given bounds on the uncertainty. This information will subsequently be used to coordinate the pursuers' motion as detailed in Section 4. Before describing the reachable set estimation, we provide some preliminaries on mixed monotonicity.

### 3.2 Mixed Monotone Reachable Set (MMRS) Computation

As described in Abate et al. (2021), we let $(x, y)$ denote the vector concatenation of $x, y \in \mathbb{R}^n$ such that $(x, y) := \left[x^\top y^\top\right]^\top \in \mathbb{R}^{2n}$. Let $\preceq$ denote the componentwise vector order i.e., $x \preceq y$ if and only if $x_i \leq y_i$ for all $i$. Given $x, y \in \mathbb{R}^n$ with $x \preceq y$,

$$[x, y] = \{z \in \mathbb{R}^n | x \preceq z \text{ and } z \preceq y\} \tag{3}$$

denotes the hyperrectangle defined by the endpoints $x$ and $y$. Let $a = (x, y) \in \mathbb{R}^{2n}$ with $x \preceq y$, we denote the hyperrectangle formed by the first and last $n$ components of $a$ as $[[a]]$ i.e., $[[a]] := [x, y]$.

Consider the system

$$\dot{x} = F(x, w) \tag{4}$$

with state $x \in \mathcal{X} \subset \mathbb{R}^n$ and time-varying disturbance input $w(t) \in \mathcal{W} = [\underline{w}\, \overline{w}] \subset \mathbb{R}^m$ where the vector field $F$ is locally Lipschitz continuous in its inputs, and that disturbance signals $w : \mathbb{R} \to \mathcal{W}$ is piece-wise continuous. The set of possible disturbances is a hyperrectangle

*Definition (Abate et al. (2021)):* Given a locally Lipschitz continuous function $d : \mathcal{X} \times \mathcal{W} \times \mathcal{X} \times \mathcal{W} \to \mathbb{R}^n$ the system (4) is *mixed-monotone with respect to d* if

- For all $x \in \mathcal{X}$ and all $w \in \mathcal{W}$ we have $d(x, w, x, w) = F(x, w)$
- For all $i, j \in \{1, ..., n\}$, with $i \neq j$, we have $\frac{\partial d_i}{\partial x_j}(x, w, \hat{x}, \hat{w}) \geq 0$ for all $(x, w, \hat{x}, \hat{w}) \in \mathcal{T}$ such that $\frac{\partial d}{\partial x_j}$ exists
- For all $i, j \in \{1, ..., n\}$, we have $\frac{\partial d_i}{\partial \hat{x}_j}(x, w, \hat{x}, \hat{w}) \leq 0$ for all $(x, w, \hat{x}, \hat{w}) \in \mathcal{T}$ such that $\frac{\partial d}{\partial \hat{x}_j}$ exists
- For all $i \in \{1, ..., n\}$ and all $k \in \{1, ..., m\}$, we have $\frac{\partial d_i}{\partial w_k}(x, w, \hat{x}, \hat{w}) \leq 0 \leq \frac{\partial d_i}{\partial \hat{w}_k}(x, w, \hat{x}, \hat{w})$

If (4) is mixed monotone with respect to $d$ then $d$ is called a *decomposition function* for (4).
We can use a mixed monotone decomposition to over-approximate the reachable set by constructing a deterministic embedding system. Given $d$,

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = E(x, \hat{x}) = \begin{bmatrix} d(x, \underline{w}, \hat{x}, \overline{w} \\ d(\hat{x}, \overline{w}, x, \underline{w}) \end{bmatrix} \tag{5}$$

We refer to (5) as the *embedding system relative to d* and $E$ the *embedding function relative to d*. We denote $\Phi^E(t, a)$ the state of (5) reached at time $t$ when beginning at state $a \in \mathcal{X} \times \mathcal{X}$ at time 0.

*Proposition (Abate et al. (2021)):* Let (4) be mixed monotone with respect to $d$ and consider $[[a]] \subset \mathcal{X} \times \mathcal{X}$. If $\Phi^E(\tau, a) \in \mathcal{X} \times \mathcal{X}$ for all $0 \leq \tau \leq t$, then $R^F(t; [[a]]) \subseteq [[\Phi^E(t, a)]]$.

This provides an efficient algorithm to overestimate the evader's reachable set for (4). We now have the machinery to create a decomposition function for the particular ballistic evader that we are considering

### 3.3 Ballistic Decomposition Function

We now provide a mixed monotone decomposition function for (2). When performing mixed monotonicity analysis it is helpful to rewrite the state variables as follows: $x = [x_1, x_2, x_3]^\top$ and $v = [x_4, x_5, x_6]^\top$.

*Proposition:* The system in (2) is mixed monotone with respect to d given by:

$$d(x, w, \hat{x}, \hat{w}) = \begin{bmatrix} x_4 \\ x_5 \\ x_6 \\ \dfrac{d^1(x, \hat{x}) + d^2(x, w, \hat{x}, \hat{w}) + d^3(x, w, \hat{x}, \hat{w})}{m} \\ \times\times \\ \times\times \end{bmatrix} \tag{6}$$

where

$$d^1(x, \hat{x}) = \begin{cases} -Tx_4\sqrt{x_4^2 + d^{sqr}(x_5, \hat{x}_5) + d^{sqr}(x_6, \hat{x}_6)} \\ \quad \text{if } -Tx_4 \geq 0 \\ -Tx_4\sqrt{x_4^2 + d^{sqr}(\hat{x}_5, x_5) + d^{sqr}(\hat{x}_6, x_6)} \\ \quad \text{if } -Tx_4 < 0 \end{cases}, \tag{7}$$

$$d^{sqr}(x, \hat{x}) = \begin{cases} x^2 & \text{if } x \geq 0 \text{ and } x \geq -\hat{x} \\ \hat{x}^2 & \text{if } x \leq 0 \text{ and } x \leq -\hat{x} \\ x\hat{x} & \text{if } x \leq 0 \leq \hat{x} \end{cases}, \tag{8}$$

$$d^2(x, w, \hat{x}, \hat{w}) = \begin{cases} \min\{Nx_5w_3, N\hat{x}_5w_3, Nx_5\hat{w}_3, N\hat{x}_5, \hat{w}_3\} \\ \quad \text{if } x \leq \hat{x} \text{ and } w \leq \hat{w} \\ \max\{Nx_5w_3, N\hat{x}_5w_3, Nx_5\hat{w}_3, N\hat{x}_5, \hat{w}_3\} \\ \quad \text{if } x > \hat{x} \text{ and } w > \hat{w} \end{cases}, \tag{9}$$

and

$$d^3(x, w, \hat{x}, \hat{w}) = \begin{cases} \min\{-Nx_6w_2, -N\hat{x}_6w_2, \\ \quad -Nx_6\hat{w}_2, -N\hat{x}_6, \hat{w}_2\} \\ \quad \text{if } x \leq \hat{x} \text{ and } w \leq \hat{w} \\ \max\{-Nx_6w_2, -N\hat{x}_6w_2, \\ \quad -Nx_6\hat{w}_2, -N\hat{x}_6, \hat{w}_2\} \\ \quad \text{if } x > \hat{x} \text{ and } w > \hat{w} \end{cases}. \tag{10}$$

The remaining entries (lines 5 and 6) in (6) are found continuing the pattern shown above using the cross-product. The decomposition function in 6 was computed using the general theory posited in Abate (2022) Chapter 3, and thus we omit a formal proof
The reachable set is computed by numerically integrating the embedding function (5). This reachable set will be used in the sequel to define the capture surface over which pursuers coordinate their motion.

## 4. PURSUER COORDINATION AND CAPTURE SURFACE

We now define the surface over which the pursuers attempt to capture the evader. From 5, we have at all times $t < T$ a conservative estimate of forward reachable set $\mathcal{E}(t)$. By the nature of the hyperrectangular initial conditions and the mechanics of the MMRS, the reachable set over-approximation is also a hyperrectangle. Because the evader $x_e \in \mathbb{R}^3$, we have $\mathcal{E}(t) \subset \mathbb{R}^3$. Let us then define the capture surface $\mathcal{C}(t)$ as the intersection of the evader's RS with the embedded subspace occupied by the pursuers, i.e., $\mathcal{C}(t) = \mathcal{E}(t) \bigcap \mathcal{M}$.

It was shown in Rivera-Ortiz et al. (2020) that a coordination strategy that allows for pursuers to persistently cover the capture surfaces suffices to guarantee capture. Though capture guarantees conditions are beyond the scope of the presented work, we provide a coverage strategy in the sequel, and reason about capture performance through Monte Carlo simulations in Section 5.

### 4.1 Coverage Control

Coverage is a technique in multi-agent control in which a group of agents attempts to spread out according to some distribution over a region in space. Let $x_i \in \mathcal{M} \subseteq \mathbb{R}^2$ be the position of the $i^{\text{th}}$ agent, $i \in \{1, ..., N\}$ in the domain of interest $\mathcal{M}$. Define a subdomain $\mathcal{S}(t) \subset \mathcal{M}$, such that agent $i$ lies in the subdomain at time $t$ if $p_i(t) \in \mathcal{S}(t)$. Let $\partial\mathcal{S}(t)$ denote the boundary of the subdomain at time $t$ and let $q(t)$ be differentiable for almost every point $q \in \partial\mathcal{S}$. We use the locational cost (Cortes et al. (2004)) as a metric of coverage performance:

$$\mathcal{H}(x(t), t) = \sum_{i=1}^{n} \int_{V_i(x(t),t)} \|x_i(t) - q\|^2 \phi(q, t) dq \tag{11}$$

where $\phi : \mathcal{S}(t) \times [0, \infty) \to (0, \infty)$ is a density function that encodes the importance of the points in a subdomain. We divide the domain into a Voronoi tessellation, given by

$$V_i(x,t) = \{q \in \mathcal{S}(t) \mid \|x_i - q\| \leq \|x_j - q\| \ \forall j\} \tag{12}$$

In Du et al. (1999) and Iri et al. (1984) it is shown that

$$\frac{\partial H}{\partial x_i} = \int_{V_i} -2(q - x_i)^\top \phi(q,t) dq \tag{13}$$

We define the mass $m_i$ and center of mass $c_i$ of the $i^{\text{th}}$ Voronoi cell $V_i$ as

$$m_i(x,t) = \int_{V_i} \phi(q,t) dq \tag{14}$$

$$c_i(x,t) = \frac{\int_{V_i} q\phi(q,t)dq}{m_i} \tag{15}$$

Thus (13) can be rewritten as

$$\frac{\partial H}{\partial x_i} = 2m_i(x_i - c_i)^\top \tag{16}$$

Where the critical points of (11) is

$$x_i(t) = c_i(x,t), \quad i = 1, \ldots, N. \tag{17}$$

When (17) is satisfied, $x = [x_1^\top, \ldots, x_N^\top]^\top$ is a centroidal Voronoi tessellation (CVT). Based on (16) the (scaled) gradient descent motion for individual agents is given by

$$\dot{x}_i = -k(x_i - c_i) \tag{18}$$

where $k$ is a positive gain. This expression is known as Lloyd's algorithm.

### *4.2 Virtual Agents*

By the properties of a mixed monotone system, the area of the hyperrectangular RS shrinks monotonically, and thus $\mathcal{C}(t)$ also shrinks monotonically. Because the dynamics of the evader are largely ballistic, the reachable set shrinks quickly at the outset of the trajectory, and stabilizes later in the trajectory. For this reason, it is advantageous for pursuer agents to continue playing, even if they fall outside of $\mathcal{C}(t)$, as they may be able to reach $\mathcal{C}(t)$ before game conclusion. Since even assuming that the pursuers begin within the capture surface $x_p \in \mathcal{C}(t)$, because $v_e \gg v_p$, there is no guarantee that they are fast enough to remain in $\mathcal{C}(t)$ as it shrinks, we are not able to use the pursuer agents positions as generators for the Voronoi tessellation needed to compute Lloyd's algorithm, as they may lie outside the domain.

For this reason, we introduce virtual agents that always lie strictly within the domain and that serve as guidance reference points for the real pursuer agents. Let each pursuer be assigned a virtual agent with position denoted $x_{vi} \in \mathcal{M}$. The virtual agent has no maximum speed limitations, and thus can always remain inside the capture surface. We propose a new control law with the virtual agents performing (a version of) Lloyd's algorithm, and the real pursuer agents moving toward the virtual agents at maximum speed.

$$\dot{x}_i = v_p \frac{(x_{vi} - x_i)}{\|x_{vi} - x_i\|}. \tag{19}$$

Some considerations for the virtual agent dynamics are discussed in the next subsection.

### *4.3 Virtual Domain*

In discrete time, the reachable set shrinks discontinuously, so even virtual agents performing coverage with unbounded velocity may be outside $\mathcal{C}(t)$ from one time step to the next. To mitigate this we consider coverage over a static virtual domain $\mathcal{D} = [0,1] \times [0,1]$. Because $\mathcal{C}(t)$ is always rectangular, we are able to find an invertible mapping that relates the real and virtual domains, $F_t : \mathcal{C}(t) \to \mathcal{D}$. Incorporating this function $F_t$ into the control law we guarantee that the virtual agents always stay in the reachable set. The dynamics for the virtual agents are now as follows:

$$\dot{x}_{vi} = F_{t+1}^{-1}(-k(F_t(x_{vi}) - F_t(c_{vi}))) \tag{20}$$

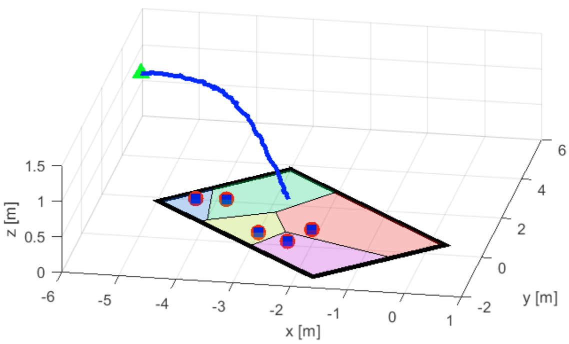
## 5. SIMULATION RESULTS

To evaluate the 3D capture strategy, we performed simulated capture of a 3D evader with planar pursuers. A dataset for real 3D ballistic trajectories were obtained using an experimental setup as follows. Unpredictable evader trajectories were obtained by lunching a Styrofoam balls from a toy ball launcher (Franklin Sports Youth Pitching Machine). This launcher imparted unpredictable initial velocity and spin (both in direction and magnitude). A representative series of trajectories was recorded using a Vicon motion capture system (Vantage V8). This data was used to generate bounds on the initial velocity and on the measured position and velocity for the reachable set computation. The controller was then implemented in MATLAB R2021B. Some representative trajectories from the simulation are shown in Fig. 2. Note how the capture surface decreases in area as the evader moves toward its target. Also notice the importance of the virtual agents. In Fig. 2b all agents are outside of the capture set $\mathcal{C}(t)$. However, by the time the game progresses to Fig. 2c, the agents are able to re-enter the capture set $\mathcal{C}(t)$ and capture the agent.

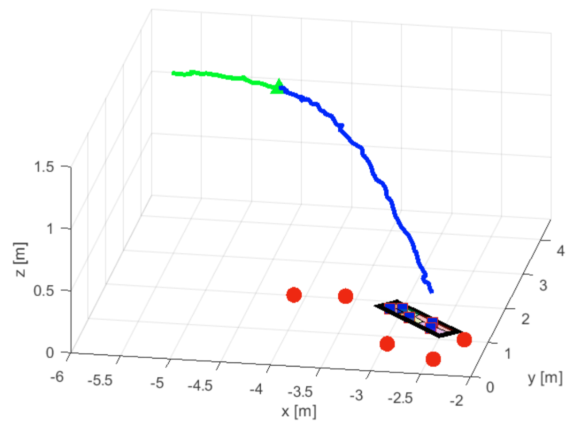
As expected, increasing the number of agents improves the probability of capture as the agents are able to coordinate their targets to better cover the RS, despite their individual kinematic disadvantage to the evader. This is conveyed by the capture statistics in Fig. 2d.

## 6. CONCLUSIONS

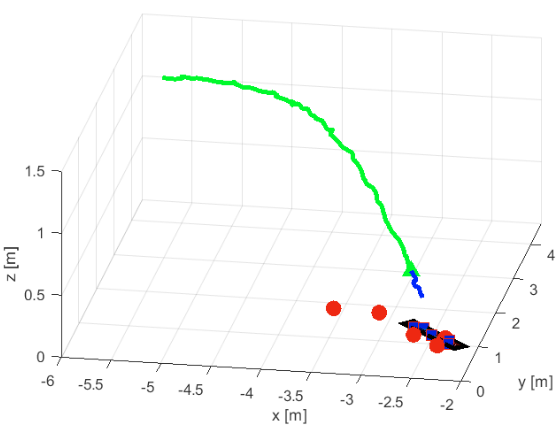
In this paper, we have presented a control strategy for a team of 2D pursuers to capture a fast, 3D, ballistic evader. This strategy leverages mixed monotone reachable set theory to overestimate the reachable set of the evader and a capture set within the subspace of the pursuers. The agents then use coverage control over a virtual domain to attempt to coordinate and capture the evader on the capture set. The algorithm was then demonstrated via numerical simulation of pursuers' motion to capture a real evader. These results show that as the number of pursuers increases, their probability of capture also increases. The coverage control strategy is also compared to a controller in which the pursuers perform pure pursuit toward the center of the capture set (Fig. 2d). Of note, pure pursuit performs better for low numbers of pursuers, but coverage performs better with higher numbers of pursuers. We suspect that



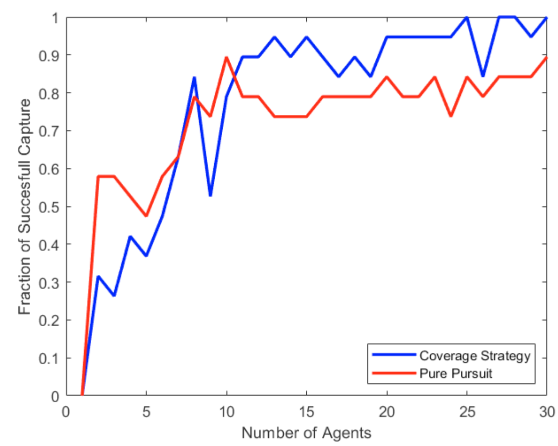

(a) Initial time estimate




(b) Coverage (midway)




(c) Capture (terminal time)




(d) Capture Statistics

Fig. 2. (a)-(c) Three snapshots from a representative game. The evader (green cone) moves along its trajectory while the pursuers (red circles) track the virtual pursuers (blue square) which perform coverage on the estimated reachable set (black rectangle). (d) The probability of capturing the evader increases with the number of agents as they are able to better cover the domain.

this is caused by the rate of contraction of the reachable set. In cases with small numbers of agents, whichever pursuers are nearest the eventual capture location are the only ones with a chance of successful capture. The pursuers adhering to pure pursuit go to the center of the reachable set. However, the pursuers adhering to coverage go to whichever Voronoi cell they were initially assigned, which may be on the other side of the reachable set. With more pursuers in the game, the coverage algorithm has enough agents to match the initial rapid contraction of the reachable set and actually perform coverage, resulting in improved performance over pure pursuit. Future work includes reasoning about the number of pursuers required to perform capture using this strategy, reasoning about the pursues' regions of dominance, and analytically exploring how evader decisions affect the reachable set in order to explicitly account for time-variations to improve pursuer tracking performance.

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